

A bootstrap test of long memory vs structural break

Un test bootstrap di memoria lunga contro l'alternativa di cambio strutturale

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Riassunto: Studi recenti hanno mostrato come un processo a memoria corta affetto da cambi di livello possa, facilmente, essere confuso con un processo a memoria lunga, in quanto entrambi presentano una funzione di autocorrelazione che decresce lentamente a zero. In questo lavoro viene proposto un test di tipo bootstrap per distinguere le due tipologie di processi. Un esperimento Monte Carlo ne illustra le *performance* in campioni finiti.

Keywords: Long memory, Occasional structural breaks, Bootstrap.

1. Introduction

Recently long memory and switching regime processes have become very popular in finance and macroeconomics. Many studies have found that processes with occasional structural breaks can produce very slowly decaying autocorrelations, which is hardly distinguished from long memory phenomenon.

In the time domain, a stationary discrete time series is said to be long memory if its autocorrelation function decays to zero like a power function. This definition implies that the dependence between successive observations decays slowly as the number of lags tends to infinity. In the frequency domain, a stationary discrete time series is said to be long memory if its spectral density is unbounded at low frequencies. In this paper we will concentrate on the fractionally integrated, or $I(d)$ processes, with $d \in (0, 1/2)$: for this range of values the process is stationary, invertible and possesses long-range dependence.

The appearance of long memory may be a genuine feature of time series or it could occur when structural breaks and regime switching are present (Granger and Hyung, 2004, Diebold and Inoue, 2001). In literature, to our knowledge, there are only a few proposals of tests to separate the long memory from the level shifts effects. Ohanissian et al. (2004) propose a test based on the invariance of the long memory parameter for temporal aggregates of the process under the null hypothesis of true long memory. Dolado, Gonzalo and Mayoral (2004) develop a test, conceived as an extension of the well-known Dickey and Fuller test, for the null hypothesis of long memory versus the alternative of $I(0)$ with deterministic components subject to structural breaks. Perron and Qu (2004) propose a test built on the difference in the log-periodogram estimate computed using different numbers of frequencies. In this framework, we propose a bootstrap test to distinguish between long memory and structural breaks. It is based on the comparison between the empirical autocorrelation of the time series and the theoretical one in case the true process is $I(d)$.

In the next section our test will be described while section 3 is devoted to our Monte Carlo experiment and concludes.

2. A bootstrap test for long memory vs structural breaks

Granger and Hyung (2004) study the sample autocorrelation function of the mean plus noise model (Chen and Tiao, 1990):

$$\begin{aligned} y_t &= m_t + \epsilon_t, & t &= 1, \dots, T, \\ m_t &= m_{t-1} + q_t \eta_t \end{aligned}$$

where ϵ_t is a noise variable and occasional level shifts, m_t , are controlled by two variables q_t (date of breaks) and η_t (size of jump). η_t is an i.i.d. $N(0, \sigma_\eta^2)$ although the normality assumption can be dropped. They prove that the sample autocorrelation function of this model can be approximated by the following expression:

$$\hat{\rho}_T(k) \approx \frac{\frac{Tp\sigma_\eta^2}{6} \left(1 - \frac{k}{T}\right) \left(1 - 2\frac{k}{T} + 4\left(\frac{k}{T}\right)^2\right)}{\frac{Tp\sigma_\eta^2}{6} + \sigma_\epsilon^2} \quad (1)$$

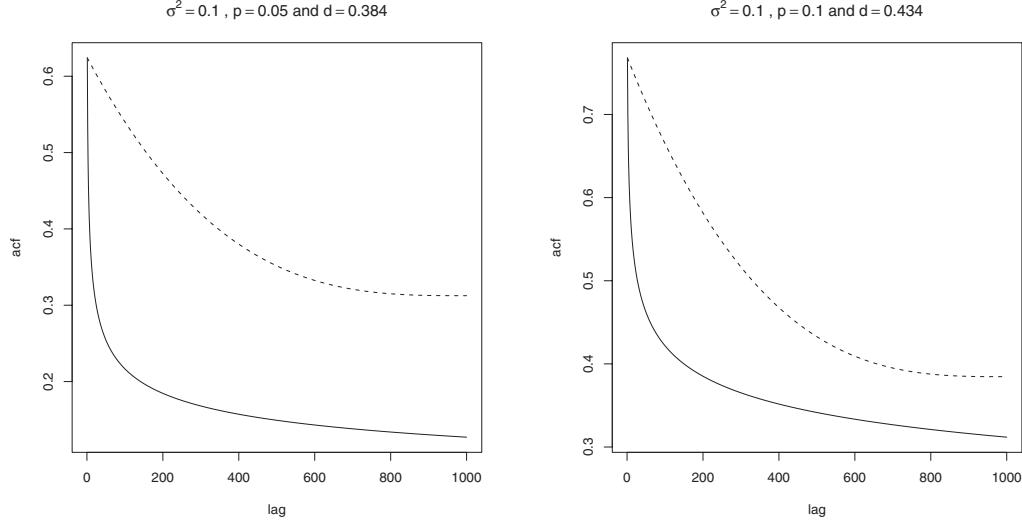
where $\approx$ denotes approximate equality for any k such that $k/T \rightarrow 0$ as T increases. The value of Tp implies an expected number of structural breaks with the sample size T , and σ_η^2 is related to the size of breaks. Moreover, the authors show that for this process, if the probability of breaks p converges to zero slowly as the sample size increases, i.e., $p \rightarrow 0$ as $T \rightarrow \infty$, then the k -th sample autocorrelation converges to nonzero value for any k such that $k/T \rightarrow 0$ as T increases. Obviously, when $0 < p < 1$ and p is fixed the break process is $I(1)$ and $\hat{\rho}_T(k) \rightarrow 1$ as $T \rightarrow \infty$, while when $p = 0$ then there are not break, the process is $I(0)$ and $\hat{\rho}_T(k) \rightarrow 0$ as $T \rightarrow \infty$. Note that, although the autocorrelations in eq. (1) decay very slowly as k increases, the asymptotic decay of the correlations of an ARFIMA process is hyperbolic, while for the occasional break process considered by Granger and Hyung (2004) it is polynomial. Moreover, the autocorrelation function for the occasional break process depends on the sample size T because the process is non stationary, while the ARFIMA process is stationary if $0 < d < 1/2$, as in our case. Thus computing the autocorrelation function of an occasional break process is misleading since we are considering just linear properties of the data. The figure 1 illustrates these important differences. In the figure the autocorrelation functions of some occasional break processes (dashed line) are compared to those of $I(d)$ processes (continuous line) that have the same first order autocorrelation, for different values of parameters.

The binomial model we described is characterized by sudden changes only. In case the structural change are gradual the class of Markov switching models (Hamilton, 1989) is more adapt. Suppose s_t is a latent random variable that can assume only values 0 or 1. s_t is assumed to be a Markov chain, with transition probability $p_{ij} = Pr(s_t = j / s_{t-1} = i)$. Then, it is possible to use a switching model for q_t such that $q_t = 0$ when $s_t = 0$ and $q_t = 1$ when $s_t = 1$. This model is a simple generalization of the previous binomial one, then we suppose that its autocorrelation function behaves like eq. (1).

The test we propose is based on a distance measure between the sample autocorrelation function of the series and the theoretical one under the hypothesis of long memory process. The test statistic is as follows

$$T = \sum_{k=1}^K (\rho(k) - \hat{\rho}(k))^2 \quad (2)$$

Figure 1: Autocorrelation for $I(d)$ and occasional break processes The continuous and dashed lines indicate $I(d)$ and occasional breaks process, respectively.



where $\rho(k)$ is the theoretical autocorrelation function of an $I(d)$ process, $\hat{\rho}(k)$ is the sample autocorrelation function, T is the series length and $K = T/4$. The distribution of our test statistic is unknown and to circumvent this problem we use a parametric bootstrap. The following is the scheme to implement our bootstrap test:

- Estimate the long memory parameter of the time series with the Whittle method, obtain $\hat{d}$ and the estimated residuals $\hat{e}_t = (1 - B)^{\hat{d}} y_t$, $t = 1, 2, \dots, T$;
- Compute the statistic T in eq. (2), with $d = \hat{d}$;
- Bootstrap parametrically the y_t , i.e. resample $\hat{e}$ (obtaining e^*) and then build $y_t^* = (1 - B)^{-\hat{d}} e_t^*$;
- For every bootstrap series y_t^* compute eq. (2), construct the empirical distribution of T and calculate the opportune percentile.

3. Monte Carlo results and conclusion

In this section we investigate with Monte Carlo experiments the power of the described test in finite samples. Simulations were conducted for the following models:

1. DGP1: mean plus noise model with $p = 0.005, 0.01, 0.05, 0.1$ and $\sigma_\eta^2 = 0.1$;
2. DGP2: Markov switching model, with $(p, q) = (0.95, 0.95; 0.95, 0.99; 0.99, 0.95; 0.99, 0.99)$ and $\sigma_\eta^2 = 0.1$.
3. DGP3: $I(d)$ model with $d = 0.1, 0.2, 0.3$ and 0.4 and $\sigma_\epsilon^2 = 1$.

Our first results are based on 300 independent replications for DGP1 and DGP2 (power of the test) and on 600 independent replications for DGP3 (size of the test). For each simulated series we considered 500 bootstrap replications and to evaluate the effects of different sample size, we have considered $T = 500, 1000$.

Table 1 presents the empirical rejection probabilities for the 5% tests. We can notice that with regards to the DGP1, the results seem to be good and the power increases with

Table 1: Estimate of power of bootstrap test, level $\alpha = 0.05$.

DGP1	p	0.005	0.01	0.05	0.1
	T=500	21.3	30.7	64.7	49.3
	T=1000	57.6	73.3	82.5	66.3
DGP2	(p,q)	$(0.95,0.95)$	$(0.95,0.99)$	$(0.99,0.95)$	$(0.99,0.99)$
	T=500	14.3	30.3	26.7	15.6
	T=1000	22.4	43.8	21.0	27.3
DGP3	d	0.1	0.2	0.3	0.4
	T=500	4.0	3.2	7.3	7.0
	T=1000	3.8	3.5	7.5	7.3

T . With regards to DGP2 the results are slightly less good, but the power tends still to increase with T . For DGP3 the percentages of rejection are encouraging, since they are relatively close to the nominal level. According to the literature, it is confirmed that the distributions of $\hat{\rho}(k)$ are different when $d < 1/4$ and $d > 1/4$. The latter is a very important point that will be considered in the authors' future work on these preliminary promising results.

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